JC03 Rec'd PCT/PTO 09 MAR 2001

U.S DEPARTMENT OF COMMERCE PATENT AND TRADEMARK OFFICE		ATTORNEY'S DOCKET NUMBER KNUDSEN 2
TRANMITTAL LETTER TO THE UNITED STATES DESIGNATED/ELECTED OFFICE (DO/EO/US) CONCERNING A FILING UNDER 35 U.S.C. 371		U S APPLICATION NO (If known, see 37 CFR 1 5) 09/786756
INTERNATIONAL APPLICATION NO. PCT/FR00/01979	INTERNATIONAL FILING DATE 07 July 2000	PRIORITY CLAIMED 09 July 1999
TITLE OF INVENTION COMPUTING METHOD FOR ELLIPTIC CURVE CRYPTOGRAPHY		
APPLICANT(S) FOR DO/EO/US **Erik KNUDSEN**		

OIPE MAR 09 2001 JC14 PATENT & TRADEMARK OFFICE

Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. [X] This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.
2. [ ] This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.
3. [X] This is an express request to begin national examination procedures (35 U.S.C. 371(f)) at any time rather than delay examination until the expiration of the applicable time limit set in 35 U.S.C. 371(b) and PCT Articles 22 and 39(1).
4. [ ] The US has been elected in a Demand by the expiration of 19 months from the priority date (PCT Article 31).
5. [X] A copy of the International Application as filed (35 U.S.C. 371(c)(2))
   a. [ ] is attached hereto (required only if not transmitted by the International Bureau).
   b. [X] has been communicated by the International Bureau.
   c. [ ] is not required, as the application was filed in the United States Receiving Office (RO/US).
6. [X] An English language translation of the International Application as filed (35 U.S.C. 371(c)(2)).
7. [X] Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371(c)(3))
   a. [ ] are transmitted herewith (required only if not transmitted by the International Bureau).
   b. [ ] have been communicated by the International Bureau.
   c. [ ] have not been made; however, the time limit for making such amendments has NOT expired.
   d. [X] have not been made and will not be made.
8. [ ] An English language translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).
9. [ ] An oath or declaration of the inventor(s) (35 U.S.C. 371(c)(4)).
10. [ ] An English language translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371(c)(5)).

Items 11. to 16. below concern document(s) or information included:

11. [X] An Information Disclosure Statement under 37 CFR 1.97 and 1.98.
12. [ ] An Assignment document for recording. A separate cover sheet in compliance with 37 CFR 3.28 and 3.31 is included.
13. [X] A FIRST preliminary amendment.
    [ ] A SECOND or SUBSEQUENT preliminary amendment.
14. [ ] A substitute specification.
15. [ ] A change of power of attorney and/or address letter.
16. [X] Other items or information:
    [X] Courtesy copy of the first page of the International Publication (WO 01/04742).
    [X] Formal drawings, 7 sheets, Figures 1-7.
    [X] Courtesy Copy of the International Search Report.

JC02 Rec'd PCT/PTO 0 9 MAR 2001

U.S. APPLICATION NO (If known,see 37 CFR 1.5) 09/786756	International Application No. PCT/FR00/01979	Attorney's Docket No KNUDSEN 2

17. [xx] The following fees are submitted:

CALCULATIONS PTO USE ONLY

BASIC NATIONAL FEE (37 CFR 1.492 (a)(1) –(5):

Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(2)) paid to USPTO and International Search Report not prepared by the EPO or JPO................**$1000.00**

International preliminary examination fee (37 CFR 1.482) not paid to USPTO but International Search Report prepared by the EPO or JPO...........**$860.00**

International preliminary examination fee (37 CFR 1.482) not paid to USPTO but international search fee (37 CFR 1.445(a)(2)) paid to USPTO....................**$710.00**

International preliminary examination fee paid to USPTO (37 CFR 1.482) but all claims did not satisfy provisions of PCT Article 33(1)-(4)................**$690.00**

International preliminary examination fee paid to USPTO (37 CFR 1.482) and all claims satisfied provisions of PCT Article 33(1)-(4).........................**$100.00**

ENTER APPROPRIATE BASIC FEE AMOUNT =				$ 860.00	
Surcharge of **$130.00** for furnishing the oath or declaration later than [X] 20 [ ] 30 months from the earliest claimed priority date (37 CFR 1.492(e)).				$ 130.00	
Claims as Originally Presented	Number Filed	Number Extra	Rate		
Total Claims	18 - 20		X $18.00	$	
Independent Claims	1 - 3		X $80.00	$	
Multiple Dependent Claims (if applicable)			+$270.00	$	
TOTAL OF ABOVE CALCULATIONS =				$ 990.00	
Claims After Post Filing Prel. Amend	Number Filed	Number Extra	Rate		
Total Claims	- 20		X $18.00	$	
Independent Claims	- 3		X $78.00	$	
TOTAL OF ABOVE CALCULATIONS =				$ 990.00	
Reduction of ½ for filing by small entity, if applicable. Applicant claims small entity status. See 37 CFR 1.27.				$	
SUBTOTAL =				$ 990.00	
Processing fee of **$130.00** for furnishing the English translation later than [ ] 20 [ ] 30 months from the earliest claimed priority date (37 CFR 1.492(f)).				$	
TOTAL NATIONAL FEE =				$ 990.00	
Fee for recording the enclosed assignment (37 CFR 1.21(h)). The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31). **$40.00** per property +				$	
TOTAL FEES ENCLOSED =				$ 990.00	
				Amount to be: refunded	$
				charged	$

a. [ ] A check in the amount of $____________ to cover the above fees is enclosed.
b. [X] Credit Card Payment Form (PTO-2038), authorizing payment in the amount of $ 990.00, is attached.
c. [ ] Please charge my Deposit Account No. **02-4035** in the amount of $____________ to cover the above fees. A duplicate copy of this sheet is enclosed.
d. [XX] The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment to Deposit Account No. **02-4035**. A duplicate copy of this sheet is enclosed.

NOTE: Where an appropriate time limit under 37 CFR 1.494 or 1.495 has not been met, a petition to revive (37 CFR 1.137(a) or (b)) must be filed and granted to restore the application to pending status.

SEND ALL CORRESPONDENCE TO:

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REGISTRATION NUMBER

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JC02 Rec'd PCT/PTO 0 9 MAR 2001

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re Application of: Erik KNUDSEN	Art Unit:
IA No.: .PCT/FR00/01979	Washington, D.C.
IA Filed: 07 July 2000	
U.S. App. No.: (Not Yet Assigned)	March 9, 2001
National Filing Date: (Not Yet Received)	
For: COMPUTING METHOD...	Docket No.: KNUDSEN 2

PRELIMINARY AMENDMENT

Honorable Commissioner for Patents and Trademarks
Washington, D.C. 20231

Sir:

Contemporaneous with the filing of this case and prior to calculation of the filing fee, kindly amend as follows:

IN THE SPECIFICATION

After the title please insert the following paragraph:

REFERENCE TO RELATED APPLICATIONS

The present application is the national stage under 35 U.S.C. §371 of international application PCT/FR00/01979, filed 07 July 2000 which designated the United States, and which application was not published in the English language.--

IN THE CLAIMS

6. A method according to claim 1, characterized in that it is a protocol for constructing a common key from two secret keys respectively belonging to the aforementioned two entities and a public key consisting of a point P of odd order of a chosen non-supersingular elliptic curve E.

8. A method according to claim 1, characterized in that it is a signature protocol between two entities based on a pair of permanent keys belonging to the one of the entities, one secret (a) and the other public (Q), resulting from the scalar multiplication of the secret key (a) by another public key consisting of a point (P) of odd order r of a chosen non-supersingular elliptic curve (E).

10. A method according to claim 7, characterized in that scalar multiplication using halvings is obtained by the following operations:

- if said scalar of the multiplication is denoted S, choose m+1 values So... Sm $\in$ {0,1} to define S as follows:

$$S = \sum_{i=0}^{m} S_i \left(\frac{r+1}{2}\right)^i$$

being the aforementioned odd order and m being the single integer between $\log_2(r) - 1$ and $\log_2(r)$,

calculate the scalar multiplication [S]P of a point P of said elliptic curve by the scalar S by applying an algorithm consisting of determining the series of points (Q_{m+1}, Q_m..., Q_i..., Q_o) of said elliptic curve E such that:

$Q_{m+1} = O$ (neutral element)

$$Q_i = [S_i]P + \left[\frac{1}{2}\right]Q_{i+1} \text{ with } o \leq i \leq m$$

calculate the last point Q_o of said series giving the result [S]P of said scalar multiplication.

In re of: Erik KNUDSEN (KNUDSEN 2)

REMARKS

The above amendment to the specification is being made to insert reference to the PCT application of which the present case is a U.S. national stage. The above amendments to the claims are being made in order to eliminate any properly multiply dependent claims, for the purpose of reducing the filing fee. Please enter this amendment prior to calculation of the filing fee in this case.

Favorable consideration is earnestly solicited.

Respectfully submitted,
BROWDY AND NEIMARK, P.L.L.C.
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In re of: Erik KNUDSEN (KNUDSEN 2)

"VERSION WITH MARKINGS TO SHOW CHANGES MADE"

6. A method according to ~~any preceding claim~~ claim 1, characterized in that it is a protocol for constructing a common key from two secret keys respectively belonging to the aforementioned two entities and a public key consisting of a point P of odd order r of a chosen non-supersingular elliptic curve E.

8. A method according to ~~any of claims 1 to 5~~ claim 1, characterized in that it is a signature protocol between two entities based on a pair of permanent keys belonging to the one of the entities, one secret (a) and the other public (Q), resulting from the scalar multiplication of the secret key (a) by another public key consisting of a point (P) of odd order r of a chosen non-supersingular elliptic curve (E).

10. A method according to claim 7~~ or claim 9~~, characterized in that scalar multiplication using halvings is obtained by the following operations:

- if said scalar of the multiplication is denoted S, choose m+1 values $S_0 \ldots S_m \in \{0,1\}$ to define S as follows:

$$S = \sum_{i=0}^{m} S_i \left(\frac{r+1}{2}\right)^i$$

r being the aforementioned odd order and m being the single integer between $\log_2(r) - 1$ and $\log_2(r)$,

calculate the scalar multiplication [S]P of a point P of said elliptic curve by the scalar S by applying an algorithm consisting of determining the series of points (Q_{m+1}, $Q_m \ldots$, $Q_1 \ldots$, Q_0) of said elliptic curve E such that:

$Q_{m+1} = O$ (neutral element)

$$Q_i = [S_i]P + \left[\frac{1}{2}\right]Q_{i+1} \text{ with } 0 \le i \le m$$

calculate the last point Q_0 of said series giving the result

[S]P of said scalar multiplication.

09/786756

7/PRTS

JC02 Rec'd PCT/PTO 09 MAR 2001

Calculation method for elliptic curve cryptography

The invention relates to a cryptographic method employed between two entities exchanging information over a non-secure communication channel, for example a cable or radio network, the method assuring the confidentiality and the integrity of information transfer between the two entities. The invention relates more particularly to an improvement to cryptosystems employing calculations on an elliptic curve. The improve mainly reduces the calculation time.

The Diffie-Hellmann key exchange cryptographic protocol is used to exchange keys securely between two entities. Using it entails employing a group in the mathematical sense of the term. A group that can be used is constituted by an elliptic curve of the following type:

$$y^2 + xy = x^3 + \alpha x^2 + \beta$$

It is known that if P = (x,y) is on the elliptic curve E, it is possible to define a "product" or "scalar multiplication" of the point P of E by an integer m. This operation is defined as follows:

$$[m]\ P = P + P + P \ldots\ldots + P \text{ (m times)}$$

Doubling a chosen point P on this kind of elliptic curve in a Diffie-Hellmann key exchange algorithm is known in the art. This operation is known as "point doubling" and is part of an iterative double-and-add process. Any such doubling takes time.

The slowest part of the Diffie-Hellman key exchange protocol is multiplying an unknown point on the curve by a random scalar. Only elliptic curves defined on a body of characteristic-two are considered here; this is a widely adopted implementation choice, because addition within a body of this kind corresponds to the "exclusive-or" operation.

It is known in the art that multiplication by a scalar can be accelerated for curves defined on a body of low cardinality by using the Frobenius morphism. The curves can be chosen so that none of the known attacks applies to them. However, it is obviously preferable, at least in principle, to be able to choose the curve to be used from a class of curves that is as general as possible. The fastest version of the method in accordance with the invention is applied to half the elliptic curves. Moreover, from a cryptographic point of view, that half is the best half. Before the theory of the method is described, the basic concepts are reviewed.

For simplicity, consider the elliptic curve (E) that can be represented geometrically and is defined for the set R of real numbers by the equation $y^2 + y = x^3 - x^2$ shown in figure 1, in which figure a horizontal line represents an integer number m, a vertical line represents an integer number n and each intersection of horizontal and vertical lines represents the integer coordinate pair (m, n).

(E) passes through a finite number of points with integer coordinates and any secant at (E) originating from any such point intersects (E) at two points, which may be coincident (in the case of tangents to the curve).

The addition operation applied to any two of these points A and B is defined as follows: let B_1 be the point at which the straight line segment (AB) intersects (E); the vertical through B_1 intersects (E) at C = A + B.

In the special case where (AB') is tangential to (E), C' is the required sum.

The "intersection of all verticals" point O is referred to as the point at infinity of (E) and is the neutral element of the addition defined in this way since, by applying the geometrical construction which defines the addition:

$A+O = O+A = A$

The doubling of A, which is denoted [2]A and defined as: A + A, is therefore the point B', the straight line segment (Ax) being tangential to (E) at A.

By applying the addition of A construction to the point B', the point [3]A is obtained, and so on: this is the definition of the product [n]A of a point by an integer.

The present invention in fact relates to a family of elliptic curves which cannot be represented geometrically but are defined as follows:

Let n be a given integer, F_{2^n} the body of 2^n elements, and $\overline{F_{2^n}}$ its algebraic closure. Let O be the point at infinity. The non-supersingular elliptic curve E defined at F_{2^n} is:

$$E = \{(x,y) \in \overline{F_{2^n}} \times \overline{F_{2^n}} \mid y^2 + xy = x^3 + \alpha x^2 + \beta\} \cup \{O\} \quad \alpha, \beta \in F^{2^n}, \beta \neq 0$$

The elements of E are usually referred to as "points". It is well known in the art that E can be given an abelian group structure by taking the point at infinity as a neutral element. Hereinafter, the finite subgroup of rational points of E is considered, and is defined by:

$$E(F_{2^n}) = \{(x,y) \in F_{2^n} \times F_{2^n} \mid y^2 + xy = x^3 + \alpha x^2 + \beta\} \cup \{O\} \quad \alpha, \beta \in F^{2^n}, \beta \neq 0$$

where N is the set of natural integers; for all $m \in N$, the "multiplication by m" application in E is defined by:

$$[m]: E \rightarrow E$$

$$P \rightarrow P + \ldots\ldots + P \text{ (m times) and } \forall P \in E: [O]P = O$$

E[m] is the kernel of the application. The points of the group E[m] are called the m-torsion points of E. The group structure of the m-torsion points is well known in the art.

In the situation in which m is a power of 2:

$$\forall k \in N: E[2^k] \cong Z/2^k Z$$

where Z is the set of relative integers.

Because $E(F_{2^n})$ is a finite sub-group of E, there exists $k' \geq 1$ such that $E[2^k]$ is contained in $E(F_{2^n})$ if and only if $k \leq k'$. For the elliptic curves E for which k'=1, the structure of $E(F_{2^n})$ is:

$$E(F_{2^n}) = G \times \{O, T_2\}$$

where G is an odd order group and T_2 designates the unique second order point of E. A curve of this kind is said to have a minimal two-torsion.

It is now possible to explain the object of the invention. Doubling is not injective when it is defined on E or $E(F_{2^n})$, because its kernel is: $E[2] = \{O, T_2\}$.

Moreover, if the domain for defining doubling is reduced to an odd order sub-group $G \subset E(F_{2^n})$ doubling becomes bijective.

As a result doubling allows an inverse application to the sub-group that is referred to hereinafter as halving:

$$[1/2]: \quad G \to G$$

$$P \to Q \text{ such that: } [2]\, Q = P$$

[1/2] P is the point of G to which the doubling application makes the point P correspond.

For all $k \geq 1$:

$$\left[\frac{1}{2^k}\right] = \left[\frac{1}{2}\right] \circ \left[\frac{1}{2}\right] \circ \ldots\ldots\ldots \circ \left[\frac{1}{2}\right]$$

represents k compositions of the halving application with itself.

Generally speaking, the invention therefore provides a cryptographic method employed between two entities exchanging information via a non-secure communication channel, the method including a step of multiplying an odd order point of a non-supersingular elliptic curve by an integer, characterized in that, for exchanging information via the non-secure communication channel, the above step includes addition and halving of

points of said elliptic curve, the addition of points is an operation known in the art, the halving of a point P is defined as the unique odd order point D such that [2]D = P, $\left[\frac{1}{2}\right]$ denotes the halving operation and $\left[\frac{1}{2}\right]$P denotes the point D.

The halving application is beneficial for the scalar multiplication of a point on an elliptic curve for the following reason: if affine coordinates are used, it is possible to replace all doublings of a point of a scalar multiplication by halvings of a point.

The halving of a point is much faster to calculate that its doubling. From a cryptographic point of view it is good to be able to choose from the greatest possible number of curves and a curve is usually used for which the two-torsion of $E(F_{2^n})$ is minimal or isomorphic to Z/4Z. For a given curve F_{2^n} the minimal two-torsion elliptic curves constitute exactly half of the set of elliptic curves defined on F_{2^n}. This is why, although it is not totally general, the fastest version of the method described applies to a good proportion of the curves in interest in cryptography. It can also be applied when the elements of the body are represented in a normal basis. In the case of a polynomial basis, the memory space required is of the order of $O(n^2)$ bits.

Some examples are given hereinafter, with reference to the accompanying drawings, in which:

- figure 1 is a graph showing a very particular elliptic curve that can be represented geometrically and is used hereinafter to explain elementary operations employed in the context of the invention;

- figure 2 is a diagram showing exchanges of information in accordance with the invention between two entities;

- figures 3 to 6 are flowcharts explaining some applications conforming to the invention; and

- figure 7 is a block diagram of another system for exchanging information between two entities A and B which can employ a cryptographic method according to the invention.

We will show how to calculate [1/2] $P \in G$ from $P \in G$. We will then show how to replace the doublings of points by halvings to execute a multiplication by a scalar.

We will use the usual affine representation of a point: $P=(x,y)$ and the representation: (x, λ_p) with $\lambda_p = x+y/x$.

We derive $y = x\ (x + \lambda_p)$, which uses only one multiplication, from the second representation.

By proceeding in this way, to multiply a point by a scalar, we save on multiplications by calculating intermediate results using the representation (x, λ_p) and the coordinate of the affine representation is determined only at the end of the calculation.

A point P is halved in the following manner:

Calculate [1/2] P from P. For this consider the two points of E:

$$P = (x,y) = (x,\ x\ (x + \lambda_p))$$

$$\text{and } Q = (u,v) = (u,\ u\ (u + \lambda_Q))$$

such that: $[2]Q = P$

The formulas for doubling known in the art yield:

$$\lambda_Q = u + v/u \qquad (1)$$

$$x = \lambda_Q^2 + \lambda_Q + \alpha \qquad (2)$$

$$y = (x+u)\ \lambda_Q + x + v \qquad (3)$$

Multiplying (1) by u and inserting the value of v obtained in this way in (3), the above system becomes:

$$v = u\ (u + \lambda_Q)$$

$$\lambda_Q^2 + \lambda_Q = \alpha + x$$

$$y = (x + u)\,\lambda_Q + x + u^2 + u\,\lambda_Q = u^2 + x\,(\lambda_Q + 1)$$

or, since $y = x\,(x + \lambda_p)$:

$$\lambda_Q^{\,2} + \lambda_Q = \alpha + x \qquad \text{(i)}$$
$$u^2 = (x\,(\lambda_Q + 1) + y = (\lambda_Q + \lambda_p + x + 1) \qquad \text{(ii)}$$
$$v = u(u + \lambda_Q) \qquad \text{(iii)}$$

Starting from $P = (x,y) = (x,\ x\,(x + \lambda_p))$ in affine coordinates or in the $(x,\ \lambda_p)$ representation, the above system of equations determines the following two types:

$$[1/2]\ P \in G \text{ and } [1[/2]\ P + T_2 \in E(F_{2^n}) \setminus G$$

which give P by doubling. The following property enables it to be distinguished.

Let E be a minimal two-torsion elliptic curve and $P \in E(F_{2^n}) = G \times \{O,\ T_2\}$ one of its odd order elements. Let $Q \in \{[1/2]\ P,\ [1/2]\ P + T_2\}$ and let Q_1 be one of the two points of E such that $[2]Q_1 = Q$.

We have the necessary and sufficient condition:

$$Q + [1/2]P \Leftrightarrow Q_1 \in E(F_{2^n}) \qquad \text{(a)}$$

We deduce from this that it is possible to check if $Q = [1/2]\ P$ by applying the formulas (i), (ii) and (iii) to Q and verifying if one of the points obtained belongs to $E(F_{2^n})$.

We can extend this process to an elliptic curve $E\ (F_{2^n}) = G \times E\ [2^k]$ that is arbitrary by applying the formulas (i), (ii) and (iii) k times: the first time to Q, to obtain a point Q_1 such that $[2]\ Q_1 = Q$; the ith time to Q_{i-1} to obtain a point Q_i such that $[2]\ Q_i = Q_{i-1}$. The resultant point Q_k will be of the form:

$\left[\frac{1}{2^{k+1}}\right] P + T_{2^{k+1}}$ if and only if $Q = [1/2]P + T_2$ and will be of the form:

$\left[\frac{1}{2^{k+1}}\right] P + T_{2^i}$ with $0 \le i \le k$ if and only if $Q = [1/2]P$. We therefore have the necessary and sufficient condition:

$$Q = [1/2]P \Leftrightarrow Q_K \in E(F_{2^n})$$

This process is evidently lengthy if k is large.

The above equation (a) shows that we can determine whether $Q = [1/2]P$ or $Q = [1/2]P + T_2$ by examining if the coordinates of Q_1 belong to F_{2^n} or to a super-body of F_{2^n}. As Q_1 is determined by the equations (i), (ii) and (iii), we have to study the operations used in solving these equations, which are not internal to the body but have their result on a super-body of F_{2^n}. The only possible instance is that of solving the second degree equation (i): we must also calculate a square root to calculate the first coordinate of Q_1, but in characteristic-two finding the square root is an operation internal to the body. Thus:

$$Q = (u, v) = [1/2]\ P \Leftrightarrow \exists \lambda \in F_{2^n} : \lambda^2 + \lambda = \alpha + u$$

Because finding the square root is internal to the body, this necessary and sufficient condition can also be written:

$$Q = (u, v) = [1/2]\ P \Leftrightarrow \exists \lambda \in F_{2^n} : \lambda^2 + \lambda = \alpha^2 + u^2$$

The preceding relation is used to optimize the algorithm referred to below in instances where the square root calculation time is not negligible.

For $P \in G$, the two solutions of (i) are $\lambda_{[1/2]P}$ and $\lambda_{[1/2]P} + 1$ and we deduce from (ii) that the first coordinates of the associated points are u and $(u + \sqrt{x})$. We can therefore deduce an algorithm for calculating $[1/2]P$ in the following manner:

If F_{2^n} is a finite body of 2^n elements, $E(F_{2^n})$ is the sub-group of an elliptic curve E defined by:

$$E(F_{2^n}) = \{(x,y) \in F_{2^n} \times F_{2^n} \mid y^2 + xy = x^3 + \alpha x^2 + \beta\} \cup \{O\}\ \alpha,$$
$$\beta \in F_{2^n},\ \beta \neq O,$$

and $E[2^k]$ is the set of points P of said elliptic curve such that P added 2^k times to itself gives the neutral element O when k is an integer greater than or equal to 1 then a point $P = (x,y)$ of said elliptic curve yields by

said halving the point $\left[\frac{1}{2}\right]$ P = (u_0, v_0) of said elliptic curve, obtained by effecting the following operations illustrated by the figure 3 flowchart:

- seek a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_0^2 such that $u_0^2 = x\ (\lambda_0 + 1) + y$
- if k has the value 1, check if the equation: $\lambda^2 + \lambda = \alpha^2 + u_0^2$ has solutions in F_{2^n},
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0^{\,2}}$$

$$v_0 = u_0\ (u_0 + \lambda_0)$$

$$\text{and}\quad \left[\frac{1}{2}\right] P = (u_0,\ v_0)$$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;
- if k is greater than 1, perform the following iterative calculation:

seek a value λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

then calculate the value u^2_i such that $u^2_i = u_{i-1}\ (\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

by incrementing i from i=1 until the value u_{i-1}^2 is obtained

- check whether the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{i-1}$ has solutions in F_{2^n}
- if so, calculate said halving is as follows:

$$u_0 = \sqrt{u_0^{\,2}}$$

$$v_0 = u_0\ (u_0 + \lambda_0)$$

$$\text{and}\quad \left[\frac{1}{2}\right] P = (u_0,\ v_0)$$

- if not, add x to the second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

If we choose to represent the point $\left[\frac{1}{2}\right]P = (u_0, v_0)$ of the elliptic curve by (u_o, λ_0) with $\lambda_0 = u_0 + v_0/u_0$, then the algorithm conforms to the figure 4 flow chart:

- seek a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_o^2 such that $u_o^2 = x(\lambda_0 + 1) + y$,
- if k has the value 1, check if the equation: $\lambda^2 + \lambda_0 = \alpha^2 + u^2_0$ has solutions in F_{2^n},
- if so, calculate said halving as follows:

  $u_0 = \sqrt{u_0^2}$

  and: $\left[\frac{1}{2}\right]P = (u_0, \lambda_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;
- if k is greater than 1 perform the following an iterative calculation:

  seek a value λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

  then calculate the value u_i^2 such that $u_i^2 = u_{i-1}(\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

incrementing i from i=1 until the value u^2_{k-1} is obtained

- check if the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{k-1}$ has solutions in F_{2^n}
- if so, calculate said halving as well as follows:

  $u_0 = \sqrt{u_0^2}$

  and $\left[\frac{1}{2}\right]P = (u_0, \lambda_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

If we choose to represent the point $P = (x,y)$ by (x, λ_p) setting $\lambda_p = x+y/x$ which gives by said halving

the point $\left[\frac{1}{2}\right]P = (u_0, v_0)$ of said elliptic curve, then the algorithm conforms to the figure 5 flow chart:

- seek a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_o^2 such that $u_o^2 = x\ (\lambda_0 + \lambda_p + x + 1)$
- if k has the value 1, check if the equation: $\lambda^2 + \lambda = \alpha^2 + u_0^2$ has solutions in F_{2^n},
- if so, calculate said halving as follows:

  $u_0 = \sqrt{u_0^2}$

  $v_o = u_o\ (u_o + \lambda_0)$

  and: $\left[\frac{1}{2}\right]P = (u_0,\ v_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;
- if k is greater than 1 perform the following an iterative calculation:

  seek a value λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

  then calculate the value u^2_i such that $u^2_i = u_{i-1}\ (\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

  incrementing i from i=1 until the value u^2_{k-1} is obtained
- check if the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{k-1}$ has solutions in F_{2^n}
- if so, calculate said halving as well as follows:

  $u_0 = \sqrt{u_0^2}$

  $v_o = u_o\ (u_o + \lambda_0)$

  and $\left[\frac{1}{2}\right]P = (u_0,\ v_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

Finally, if we choose to represent the point P= (x,y) by (x, λ_p) with

$\lambda_p = x + y/x$ which gives by said halving the point $\left[\frac{1}{2}\right]P =$ (u_0, v_0) of the elliptic curve represented by (u_0, λ_0) with $\lambda_0 = u_0 + v_0/u_0$ then the algorithm conforms to the figure 6 algorithm:

- seek a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_o^2 such that $u_o^2 = x\ (\lambda_0 + \lambda_p + x + 1)$,
- if k has the value 1 check if the equation $\lambda^2 + \lambda = \alpha^2 + u_0^2$ has solutions in F_{2^n},
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0^2}$$

and $\left[\frac{1}{2}\right]P = (u_0, \lambda_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;
- if k is greater than 1 perform the following iterative calculation:

seek a value λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

then calculate the value u^2_i such that $u_i^2 = u_{i-1}\ (\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

incrementing i from i=1 until the value u^2_{k-1} is obtained

- check if the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{k-1}$ has solutions in F_{2^n}
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0^2}$$

and $\left[\frac{1}{2}\right]P = (u_0, \lambda_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

We next describe how to perform the check, solve the second degree equation and calculate the square root

in the algorithm for halving a point rapidly. We consider the normal basis and the polynomial basis.

The normal basis results are known in the art. We can consider F_{2^n} as the n-dimensional vectorial space on F_2. In a normal basis, an element of the body is represented by:

$$x = \sum_{i=0}^{n-1} x_i \beta^{2^i} \qquad x_i \in \{0,1\}$$

where $\beta \in F_{2^n}$ is chosen such that: $\{\beta, \beta^2, \ldots, \beta^{2^{n-1}}\}$ is a basis F_{2^n}. In a normal basis, the square root is calculated by a left circular shift and squaring is effected by a right circular shift. The corresponding calculation times are therefore negligible.

If the second degree equation: $\lambda^2 + \lambda = x$ has its solutions in F_{2^n}, a solution is then given by:

$$\lambda = \sum_{i=1}^{n-1} \lambda_i \beta^{2^i} \quad \text{with: } \lambda_i = \sum_{k=1}^{i} x_i \qquad 1 \leq i \leq n-1$$

The time to calculate λ is negligible compared to the time to calculate a multiplication of an inversion in the body. As the time to calculate a solution of the second degree equation is negligible, the check can be effected as follows: calculate a candidate λ from x and check if $\lambda^2 + \lambda = x$. If not, the equation has no solution in F_{2^n}.

In a polynomial basis, the following representation is used:

$x = \sum_{i=0}^{n-1} x_i T^i$ with $x_i \in \{0,1\}$. The square root of x can be calculated by storing the element $\sqrt{T}$ if we note that:

- in a body of characteristic-two, the square root is a morphism of the body,

$$\sqrt{\sum_{i \text{ even}} x_i T^i} = \sum_{i \text{ even}} x_i T^{\frac{i}{2}}$$

Grouping in x the even and odd powers of T and taking the square root, this becomes:

$$\sqrt{x} = \sum_{i \text{ even}} x_i T^{\frac{i}{2}} + \sqrt{T} \sum_{i \text{ odd}} x_i T^{\frac{i-1}{2}}$$

so that, to calculate a square root, it is sufficient to "reduce" two vectors by half and therefore to execute a multiplication of a previously calculated value by an element of length n/2. This is why the time to calculate a square root in a polynomial basis is equivalent to half the time to calculate a multiplication in the body.

For the check and for solving the second degree equation, we consider F_{2^n} as a n-dimensional vectorial space on F_2. The application F defined as follows:

$$F : F_{2^n} \rightarrow F_{2^n}$$
$$\lambda \rightarrow \lambda^2 + \lambda$$

is then a linear kernel operator {0, 1}

For a given x, the equation $\lambda^2 + \lambda = x$ has its solutions in F_{2^n} if and only if the vector x is in the image of F. Im(F) is an (n - 1)-dimensional sub-space of F_{2^n}. For a given basis of F_{2^n} and the corresponding scalar product there exists a single non-trivial vector orthogonal to all the vectors of Im(F). Let w be that vector. We have:

$$\exists \lambda \in F_{2^n} : \lambda^2 + \lambda = x \Leftrightarrow x \bullet w = 0$$

Accordingly, the check can be performed by adding the components of x to which components of w equal to 1 correspond. The time to perform this check is negligible.

To solve the second degree equation: $F(\lambda) = \lambda^2 + \lambda = x$ in a polynomial basis, we propose a simple and direct method which imposes the storage of an n x n matrix. For this we look for a linear operator G such that:

$$\forall x \in \mathrm{Im}(F) : F(G(x)) = (G(x))^2 + G(x) = x$$

Let $\gamma \in F_{2^n}$ be a vector such that $\gamma \notin \mathrm{Im}(F)$ and define G as follows:

$$G = \tilde{F}^{-1} \quad \text{with} \quad \tilde{F}(T^i) = \begin{cases} \gamma & \text{if: } i = 0 \\ F(T^i) & \text{if: } 1 \le i \le n\text{-}1 \end{cases}$$

Given that $x = \sum_{i=1}^{n-1} x_i F(T^i) \in \mathrm{Im}(F)$ then G(x) is a solution of the second degree equation. One implementation consists of precalculating the matrix representing G in the basis $\{1, T, \ldots\ldots, T^{n-1}\}$. In characteristic-two, the multiplication of a matrix by a vector is reduced to adding columns of the matrix to which a component of the vector equal to 1 corresponds. It follows that this method of solving a second degree equation consumes on average n/2 additions in the body F_{2^n}.

Application of the principles explained above to scalar multiplication is described below.

Let $P \in E(F_{2^n})$ be a point of odd order r, c a random integer and m the integer part of $\log_2$ (r). We calculate the product [c]P of a point by a scalar using the application for halving a point.

We show that:

For any integer c, there is a rational number of the form:

$$\sum_{i=0}^{m} \frac{c_i}{2^i} \qquad c_i \in \{0,1\}$$

such that:

$$c \equiv \sum_{i=0}^{m} \frac{c_i}{2^i} \pmod r$$

Let <P> be the cyclic group generated by P. Because of the ring isomorphism:

$$\begin{array}{ccc} P & \approx & \mathbf{Z}/r\mathbf{Z} \\ [k]P & \rightarrow & k \end{array}$$

The scalar multiplication can be calculated as follows:

$$[c]P = \sum_{i=0}^{m} \left[\frac{c_i}{2}\right] P$$

using halving and addition. We can use the double-and-add algorithm well known in the art for these calculations. For that it is sufficient to replace doubling by halving in the algorithm. It is necessary to execute $\log_2$ (r)

halvings and, on average, $1/2 \log_2 (r)$ additions. There are improved versions of the double-and-add algorithm which require only $1/3 \log_2 (r)$ additions on average.

Consequently, a scalar multiplication using a halving as defined above is obtained by means of the following operations:

- if said scalar of the multiplication is denoted S, choose m+1 values

$S_0 \ldots S_m \in \{0,1\}$ to define S as follows:

$$S = \sum_{i=0}^{m} S_i \left(\frac{r+1}{2} \right)^i$$

- r being the aforementioned odd order and m being the single integer between $\log_2(r) -1$ and $\log_2(r)$,

- calculate the scalar multiplication [S]P of a point P of said elliptic curve by the scalar S by applying an algorithm consisting of determining the series of points $(Q_{m+1}, Q_m \ldots, Q_1 \ldots, Q_o)$ of said elliptic curve E such that:

$Q_{m+1} = O$ (neutral element)

$Q_i = [S_i]P + \left[\frac{1}{2}\right] Q_{i+1}$ with $o \le i \le m$

- calculate the last point Q_o of said series giving the result [S] P of said scalar multiplication.

To add the initial point P to an intermediate result $Q = \left[\frac{1}{2}\right] Q_i$, we use the following algorithm, which is a slightly modified version of the standard algorithm:

Input: $P = (x,y)$ in affine coordinates and $Q = (u, u(u + \lambda_Q))$ represented by (u, λ_Q)

Output: $P + Q = (s, t)$ in affine coordinates

algorithm:

1. Calculate: $\lambda = \dfrac{y + u(u+\lambda_Q)}{x+u}$
2. Calculate: $s = \lambda^2 + \lambda + a + x + u$
3. Calculate: $t = (s+x)\lambda + s + y$

4. Result: (s, t)

This algorithm uses one inversion, three multiplications and one square root.

Much time is saved by replacing doubling by halving. In affine coordinates, doubling and addition both require: one inversion, two multiplications and a square root. If the scalar of the multiplication by a scalar is represented by a bit vector of length m and of k non-zero components, scalar multiplication requires:

operation	double and add	halve and add
inversions	m + k	k
multiplications	2m + 2k	m + 3k
squarings	m + k	k
solutions of $\lambda^2+\lambda=a+x$	0	m
square roots	0	m
checks	0	m

Thus using halving saves m inversions, m-k multiplications and m squarings at the cost of adding m second degree solutions, m square roots and m checks.

In a polynomial basis, an execution time improvement of around 50% can be obtained.

In a normal basis, we estimate the time to calculate the square root, perform the check and solve the second degree equation negligible compared to the time to calculate a multiplication or an inversion. Assuming further that the time to calculate an inversion is equivalent to the time to calculate three multiplications, we arrive at an execution time improvement of 55%.

Figure 2 is a diagram showing one possible application of the algorithms described above between two

entities A and B exchanging information over a non-secure communication channel. Said communication channel can consist of simple electrical connections established between the two entities for the time of a transaction. It can also include a radio and/or optical telecommunication network. In this instance the entity A is a microcircuit card and the entity B is a server. Once connected to each other via said communication channel, the two entities apply a common key construction protocol. For this purpose:

- entity A has a secret key a
- entity B has a secret key b

They must generate a secret key x known only to them from a public key consisting of a point P of odd order r of a chosen non-supersingular elliptic curve E.

The protocol employed is a Diffie-Hellman protocol, substituting for the usual "multiplication-by-two" referred to as doubling the operation in accordance with the invention described above and referred to as "halving".

The algorithm for this is as follows:

- the first entity (for example A) calculates the scalar multiplication [a]P and sends the result point to the second entity,

- the second entity (B) calculates the scalar multiplication [b]P and sends the result point to the first entity,

- the two entities respectively calculate a common point (C) = (x,y) of said elliptic curve (E) by respectively effecting the scalar multiplications [a]([b]P) and [b]([a]P), both equal to [a.b]P, and

- the two entities choose as their common key the coordinate x of said common point (C) obtained by said scalar multiplication [a.b]P, at least one of the

preceding scalar multiplications, and preferably all of them, being effected by means of predefined halvings.

To give a more precise example of this, figure 7 shows a server B connected to a communication network 1 via a communication interface 2, for example a modem interface. Similarly, a calculation station 3 is connected to the network 1 via a communication interface 4. The station 3 is equipped with a microcircuit card reader 5 into which the microcircuit card A is inserted.

The random access memory 6 of the server B contains a program 7 capable of executing cryptographic calculations on elliptic curves and in particular the product of a point by a scalar and the halving of a point.

The card A contain a central processor unit 11, a random access memory (RAM) 8, a read-only memory (ROM) 9 and an electrically erasable programmable read-only memory (EEPROM) 10. One of the memories 9 or 10 contains a program 12 capable of executing cryptographic calculations on elliptic curves and in particular the product of a point by a scalar and the halving of a point.

The two programs 7 and 12 have a common reference consisting of the same elliptic curve (E) and the same point $P=(x_0, y_0)$ of (E).

When A wishes to construct in parallel with B a common secret key for securing dialog with B, it chooses a scalar a and sends to B the product $Q=[a]P=(x_1, y_1)$. In response to this, B chooses a scalar b and sends back to A the product $R=[b]P = (x_2, y_2)$.

A then calculates the product $[a]\ R = [ab]P = (x, y)$ and B calculates the product $[b]\ Q = [ab]P = (x, y)$ and A and B adopt x as a common secret key.

These operations are represented in the table below. Those which are effected in the server B are

indicated in the right-hand column and those which are effected in the card A are indicated in the left-hand column. The horizontal arrows symbolize transfers of information via the network 1.

A		B
$P = (x_0, y_0)$		$P = (x_0, y_0)$
choice a		
$Q = [a]P = (x_1, y_1)$	$\longrightarrow$	
		choice b
	$\longleftarrow$	$R = [b]\ P = (x_2, y_2)$
$[a]R = (x, y)$		$[b]\ Q = (x,y)$
key = x		key = x

Another application of the invention applies between the two entities A and B in figure 7. It consists of a protocol for signing a message M transmitted between A and B via the non-secure channel, i.e. the network 1. The object of this protocol, the broad outlines of which are known in the art, is to make it certain that the message received by one entity was sent by the other entity.

To this end, the sending entity (for example A) has two permanent keys, namely a secret key <u>a</u> and a public key Q = [a] P, P being a point on an elliptic curve (E), and P and (E) being known to and agreed on by A and B. Another public key is the point P of odd order r of the chosen non-supersingular elliptic curve E. The operations effected entail halvings in the sense defined above.

In one example:

- the first entity (A) holding said pair of permanent keys constructs a single-use pair of keys, one key (g) chosen arbitrarily and the other key [g] P resulting from scalar multiplication of said arbitrarily

chosen key (g) by the public point P of said elliptic curve, the coordinates of the key ([g]P) being denoted (x,y) with $2 \leq g \leq r-2$,

- the first entity (A) converts the polynomial x of said single-use key [g]P = (x,y) into an integer i whose binary value is represented by the sequence of binary coefficients of said polynomial x,

- said first entity (A) calculates a signature (c,d) of the message (M) as follows:

$c = i$ modulo r

$d = g^{-1}$ (M + ac) modulo r,

- said first entity sends said message (M) and said signature (c, d) to said second entity; on receiving it:

- said second entity (B) checks if the elements of said signature (c,d) each belong to the range [1, r-1],

- if not, it declares the signature invalid and stops

- if so, said second entity (B) calculates three parameters:

$h = d^{-1}$ modulo r

$h_1 = Mh$ modulo r

$h_2 = ch$ modulo r

- said second entity calculates a point T of said elliptic curve by summing the scalar multiplications of the points P and Q by the last two parameters cited:

$T = [h_1]\ P + [h_2]\ Q$

if the resultant point T is the neutral element, said second entity declares the signature invalid and stops.

if it is not the neutral element, considering the point T with coordinates x' and y': T = (x',y'):

- said second entity (B) converts the polynomial x' of that point into an integer i' whose binary value is represented by the sequence of binary coefficients of said polynomial x',

- said second entity (B) calculates $c' = i'$ modulo r, and:

- checks that $c' = c$: if so it validates said signature and if not it invalidates it, at least one of the scalar multiplication operations and preferably all of them being effected by means of the predefined halvings.

These operations can be represented by the table below in which the operations effected in the server B are indicated in the right-hand column and the operations effected in the card A are indicated in the left-hand column, the arrow between the two columns symbolizing the transfer of information via the network 1.

A

B

choice g $2 \leq g \leq r-2$

$[g]\ P = x,\ y$

$x = \sum x_i t^i \rightarrow i = \sum x_i 2^i$

message M

$c = i \bmod r$

$d = g^{-1}\ (M+ac) \bmod r$

$M,\ (c,\ d) \longrightarrow 1 \leq c \leq r-1$? no → error → BAD

yes ↓

$1 \leq d \leq r-1$? no → error → BAD

yes ↓

$h = d^{-1} \bmod r$

$h_1 = Mh \bmod r$

$h_2 = ch \bmod r$

$T = [h_1]\ P + [h_2]\ Q = (x',\ y')$

$T = O$? yes → BAD

no

$x' - \sum x_i t^i \rightarrow i' = \sum x_i 2^i$

$c' = i' \bmod r$

$c' = c$? no → BAD

yes ↓

GOOD

BAD

CLAIMS

1. A cryptographic method employed between two entities exchanging information via a non-secure communication channel, the method including a step of multiplying an odd order point of a non-supersingular elliptic curve by an integer, characterized in that, for exchanging information via the non-secure communication channel, the above step includes addition and halving of points of said elliptic curve, the addition of points is an operation known in the art, the halving of a point P is defined as the unique odd order point D such that [2]D = P, $\left[\frac{1}{2}\right]$ denotes the halving operation and $\left[\frac{1}{2}\right]P$ denotes the point D.

2. A method according to claim 1, where F_{2^n} is a finite body of 2^n elements, $E(F_{2^n})$ is the sub-group of an elliptic curve E defined by:

$E(F_{2^n}) = \{(x,y) \in F_{2^n} \times F_{2^n} \mid y^2 + xy = x^3 + \alpha x^2 + \beta\} \cup \{O\}\ \alpha,\ \beta \in F_{2^n},\ \beta \neq 0$

and $E[2^k]$ is the set of points P of said elliptic curve such that P added 2^k times to itself gives the neutral element O, where k is an integer greater than or equal to 1, characterized in that a point P = (x,y) of said elliptic curve gives by said halving the point $\left[\frac{1}{2}\right]P = (u_o, v_o)$ of said elliptic curve obtained by effecting the following operations:

- seek a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_o^2 such that $u_o^2 = x(\lambda_0 + 1) + y$
- if k has the value 1, check if the equation: $\lambda^2 + \lambda = \alpha^2 + u_0^2$ has solutions in F_2^n,
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0^2}$$

$$v_o = u_o\ (u_o + \lambda_0)$$

$$\text{and}\quad \left[\frac{1}{2}\right] P = (u_0,\ v_0)$$

• if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;

• if k is greater than 1, perform an iterative calculation as follows:

seek a value λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

then calculate the value u^2_i such that $u^2_i = u_{i-1}\ (\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

by incrementing i from i=1 until the value u^2_{k-1} is obtained

• check whether the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{k-1}$ has solutions in F_2^n

• if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0^{\ 2}}$$

$$v_o = u_o\ (u_o + \lambda_0)$$

$$\text{and}\quad \left[\frac{1}{2}\right] P = (u_0,\ v_0)$$

• if not, add x to the second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

3. A method according to claim 1, where F_2^n is a finite body of 2^n elements, $E(F_2^n)$ is the sub-group of an elliptic curve E defined by:

$E(F_2^n) = \{(x,y) \in F_2^n \times F_2^n\ | y^2 + xy = x^3 + \alpha x^2 + \beta\} \cup \{O\}\ \alpha,\ \beta \in F_2^n,\ \beta \neq 0$ and $E[2^k]$ is the set of points P of said elliptic curve such that P added 2^k times to itself gives the neutral element O, where k is an integer greater than or equal to 1, characterized in that a point P = (x,y) of said elliptic curve gives by said halving the point $\left[\frac{1}{2}\right]P = (u_0, \lambda_0)$ of said elliptic curve,

with $\lambda_0 = u_0 + v_0/u_0$, obtained by effecting the following operations:

- seek a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_0^2 such that : $u_0^2 = x(\lambda_0 + 1) + y$
- if k has the value 1, check if the equation : $\lambda^2 + \lambda = \alpha^2 + u_0^2$ has solutions in F_{2^n},
- if so, calculate said halving as follows :

$$u_0 = \sqrt{u_0^{\,2}}$$

and $\left[\frac{1}{2}\right]P = (u_0, \lambda_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;
- if k is greater than 1, perform the following iterative calculation:

seek a value λ_i, such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

then calculate the value u_i^2 such that $u_i^2 = u_{i-1}(\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

by incrementing i from i = 1 until the value u^2_{k-1} is obtained

- check if the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{k-1}$ has solutions in F_{2^n}
- if so, calculate said halving as folows:

$$u_0 = \sqrt{u_0^{\,2}} \quad \text{and} \quad \left[\frac{1}{2}\right]P = (u_0, \lambda_0)$$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

4. A method according to claim 1, where F_{2^n} is a finite body of 2^n elements, $E(F_{2^n})$ is the sub-group of an elliptic curve E defined by:

$$E(F_{2^n}) = \{(x,y) \in F_{2^n} \times F_{2^n} \mid y^2 + xy = x^3 + \alpha x^2 + \beta\} \cup \{O\}\ \alpha, \beta \in F_{2^n}, \beta \neq 0$$

and $E[2^k]$ is the set of points P of said elliptic curve such that P added 2^k times to itself gives the neutral element O, where k is an integer greater than or equal to 1, characterized in that a point $P = (x,y)$ of said elliptic curve represented by (x,λ_p) with $\lambda_p = x + y/x$ gives by said halving the point $\left[\frac{1}{2}\right]P = (u_0, v_0)$ of said elliptic curve obtained by effecting the following operations:

- seek a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_o^2 such that $u_o^2 = x\ (\lambda_0 + \lambda_p + x + 1)$
- if k has the value 1, check if the equation: $\lambda^2 + \lambda = \alpha^2 + u_0^2$ has solutions in $F_2{}^n$,
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0{}^2}$$

$$v_o = u_o\ (u_o + \lambda_0)$$

and: $\left[\frac{1}{2}\right]P = (u_0, v_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;
- if k is greater than 1, perform the following iterative calculation:

seek a value λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

then calculate the value u_i^2 such that $u_i^2 = u_{i-1}\ (\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

incrementing i from i=1 until the value u^2_{k-1} is obtained

- check if the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{k-1}$ has solutions in $F_2{}^n$
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0{}^2}$$

$$v_o = u_o\ (u_o + \lambda_0)$$

and $\left[\frac{1}{2}\right]P = (u_0,\ v_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

5. A method according to claim 1, where F_2^n is a finite body of 2^n elements, $E(F_2^n)$ is the sub-group of an elliptic curve E defined by:

$E(F_2^n) = \{(x,y) \in F_2^n \times F_2^n \mid y^2 + xy = x^3 + \alpha x^2 + \beta\} \cup \{O\}\ \alpha,\ \beta \in F_2^n,\ \beta \neq 0$

and $E[2^k]$ is the set of points P of said elliptic curve such that P added 2^k times to itself gives the neutral element O, where k is an integer greater than or equal to 1, characterized in that a point $P = (x,y)$ of said elliptic curve represented by (x, λ_P) with $\lambda_P = x + y/x$ gives by said halving the point $\left[\frac{1}{2}\right]P = (u_0,\ v_0)$ of said elliptic curve represented by $(u_o,\ \lambda_0)$, with $\lambda_o = u_o + v_o/u_o$ obtained by effecting the following operations:

- seek for a first value λ_0 such that $\lambda_0^2 + \lambda_0 = \alpha + x$
- calculate a second value u_o^2 such that $u_o^2 = x\ (\lambda_0 + \lambda_P + x + 1)$,
- if k has the value 1, check if the equation $\lambda^2 + \lambda = \alpha^2 + u_0^2$ has solutions in F_2^n,
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0^2}$$

and $\left[\frac{1}{2}\right]P = (u_0,\ \lambda_0)$

- if not, add x to said second value u_0^2 and 1 to said first value λ_0 to calculate said halving as in the preceding operation;
- if k is greater than 1, perform the following iterative calculation:

seek a value λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$

then calculate the value u_i^2 such that $u_i^2 = u_{i-1}\ (\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$

incrementing i from i=1 until the value u^2_{k-1} is obtained

- check if the equation $\lambda^2 + \lambda = \alpha^2 + u^2_{k-1}$ has solutions in $F_2{}^n$
- if so, calculate said halving as follows:

$$u_0 = \sqrt{u_0{}^2}$$

$$\text{and}\quad \left[\frac{1}{2}\right]P = (u_0,\ \lambda_0)$$

- if not, add x to said second value $u_0{}^2$ and 1 to said first value λ_0 to calculate said halving as in the preceding operation.

6. A method according to any preceding claim, characterized in that it is a protocol for constructing a common key from two secret keys respectively belonging to the aforementioned two entities and a public key consisting of a point P of odd order r of a chosen non-supersingular elliptic curve E.

7. A method according to claim 6, characterized in that a and b are the secret keys of first and second entities, respectively, as known in the art, and:

- the first entity calculates the scalar multiplication [a]P and sends the result point to the second entity,

- the second entity calculates the scalar multiplication [b]P and sends the result point to the first entity,

- the two entities respectively calculate a common point (C) = (x,y) of said elliptic curve (E) by respectively effecting the scalar multiplications [a] ([b]P) and [b] ([a]P), both equal to [a.b]P, and

- the two entities choose as their common key the coordinate (x) of said common point (C) obtained by said scalar multiplication [a.b]P, at least one of the

preceding scalar multiplications, and preferably all of them, being effected by means of predefined halvings.

8. A method according to any of claims 1 to 5, characterized in that it is a signature protocol between two entities based on a pair of permanent keys belonging to the one of the entities, one secret (a) and the other public (Q), resulting from the scalar multiplication of the secret key (a) by another public key consisting of a point (P) of odd order r of a chosen non-supersingular elliptic curve (E).

9. A method according to claim 8, characterized by the following operations:

- the first entity (A) holding said pair of permanent keys constructs a single-use pair of keys, one key (g) being chosen arbitrarily and the other key [g]P resulting from scalar multiplication of said arbitrarily chosen key (g) by the public point P of said elliptic curve, the coordinates of the key ([g]P) being denoted (x,y) with $2 \le g \le r-2$,

- the first entity (A) converts the polynomial x of said single-use key [g]P = (x,y) into an integer i whose binary value is represented by the sequence of binary coefficients of said polynomial x,

- said first entity (A) calculates a signature (c,d) of the message (M) as follows:

c = i modulo r

$d = g^{-1}$ (M + ac) modulo r,

- said first entity sends said message (M) and said signature (c, d) to said second entity; on receiving it:

- said second entity (B) checks if the elements of said signature (c,d) each belong to the range [1, r-1],

- if not, it declares the signature invalid and stops

- if so, said second entity (B) calculates three parameters:

$h = d^{-1}$ modulo r

h_1 = Mh modulo r

h_2 = ch modulo r

- said second entity calculates a point T of said elliptic curve by summing the scalar multiplications of the points P and Q by the last two parameters cited:

$T = [h_1]\ P + [h_2]\ Q$

if the resultant point T is the neutral element, said second entity declares the signature invalid and stops;

if not, considering the point T with coordinates x' and y': T = (x',y'),

- said second entity (B) converts the polynomial x' of that point into an integer i' whose binary value is represented by the sequence of binary coefficients or said polynomial x',

- said second entity (B) calculates c' = i' modulo r and,

- checks if c' = c, in which case it validates said signature, or if not invalidates it, at least one aforementioned scalar multiplication operation and preferably all of them being effected by means of the predefined halvings.

10. A method according to claim 7 or claim 9, characterized in that scalar multiplication using halvings is obtained by the following operations:

- if said scalar of the multiplication is denoted S, choose m+1 values So… Sm ∈ {0,1} to define S as follows:

$$S = \sum_{i=0}^{m} S_i \left(\frac{r+1}{2} \right)^i$$

r being the aforementioned odd order and m being the single integer between $\log_2(r) - 1$ and $\log_2(r)$,

calculate the scalar multiplication [S]P of a point P of said elliptic curve by the scalar S by applying an

algorithm consisting of determining the series of points (Q_{m+1}, Q_m..., Q_1..., Q_o) of said elliptic curve E such that:

$Q_{m+1} = O$ (neutral element)

$Q_i = [S_i]P + \left[\frac{1}{2}\right]Q_{i+1}$ with $o \leq i \leq m$

calculate the last point Q_o of said series giving the result [S]P of said scalar multiplication.

ABSTRACT

The invention concerns fast cryptographic method between two entities exchanging data via a non-secure communication channel. The method, for example for forming a common key between two entities (A,B) each having a secret key (a,b) and using a public key (P) formed by a point of an elliptic curve (E), comprises at least a step which consists in multiplying said odd order point (P) by an integer and said phase comprises operations called additions and halving, the latter operation characterizing the invention.

FIG.1

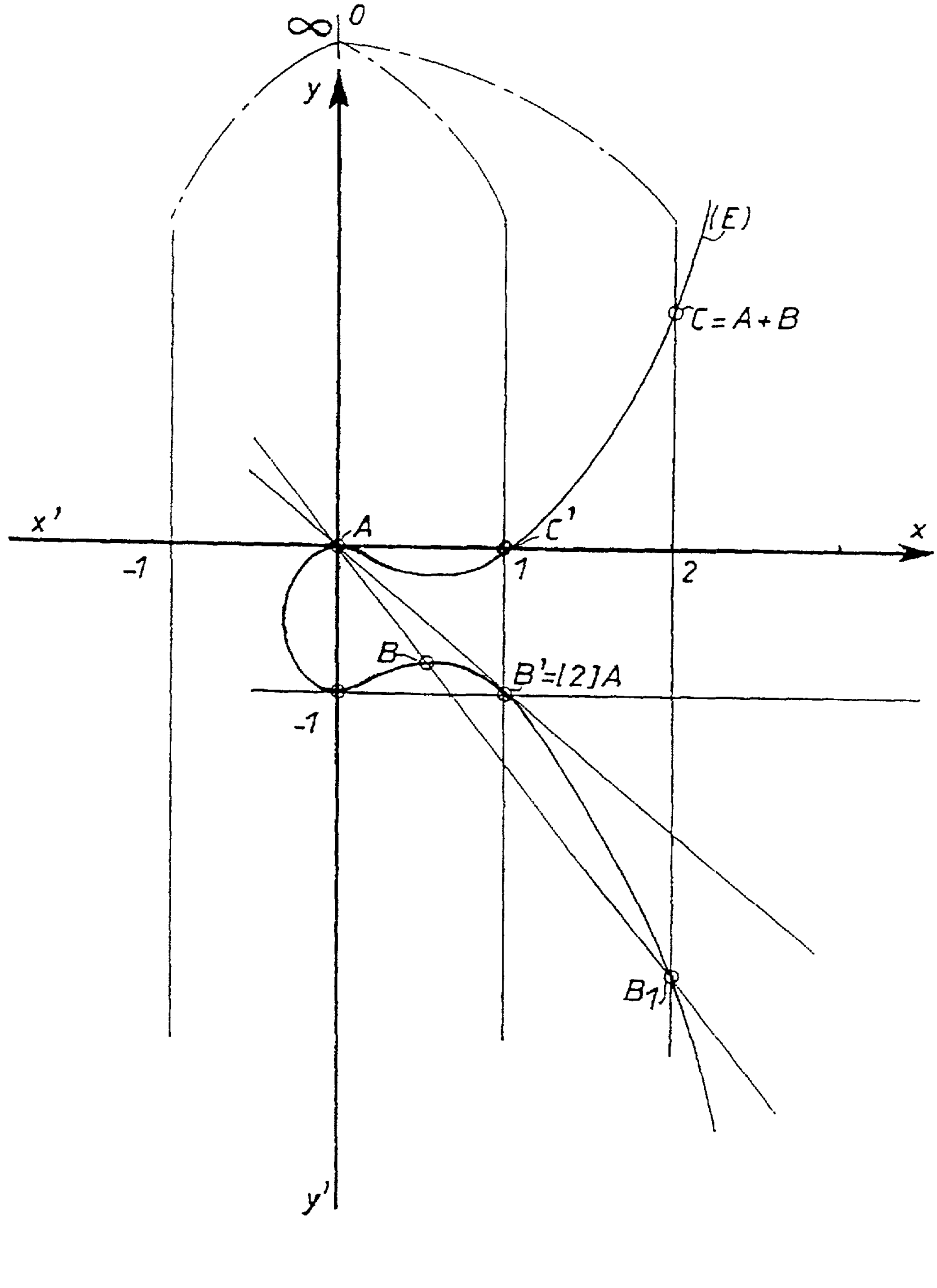


FIG. 2

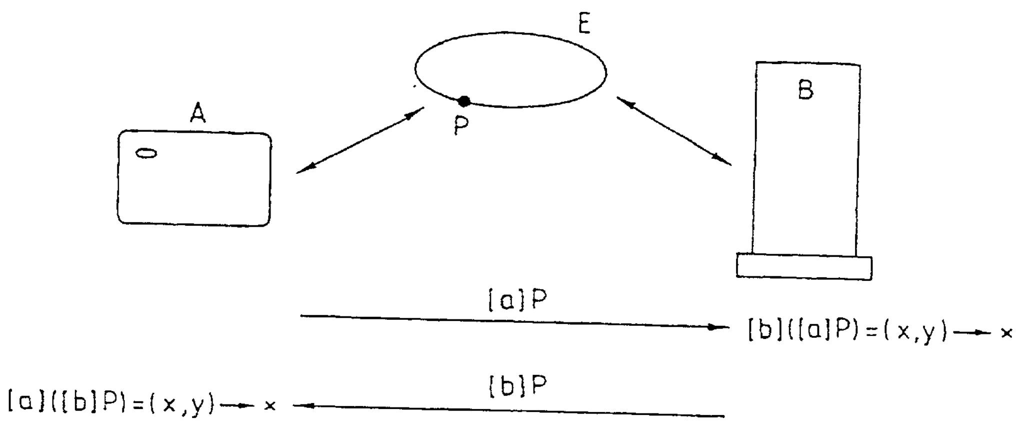
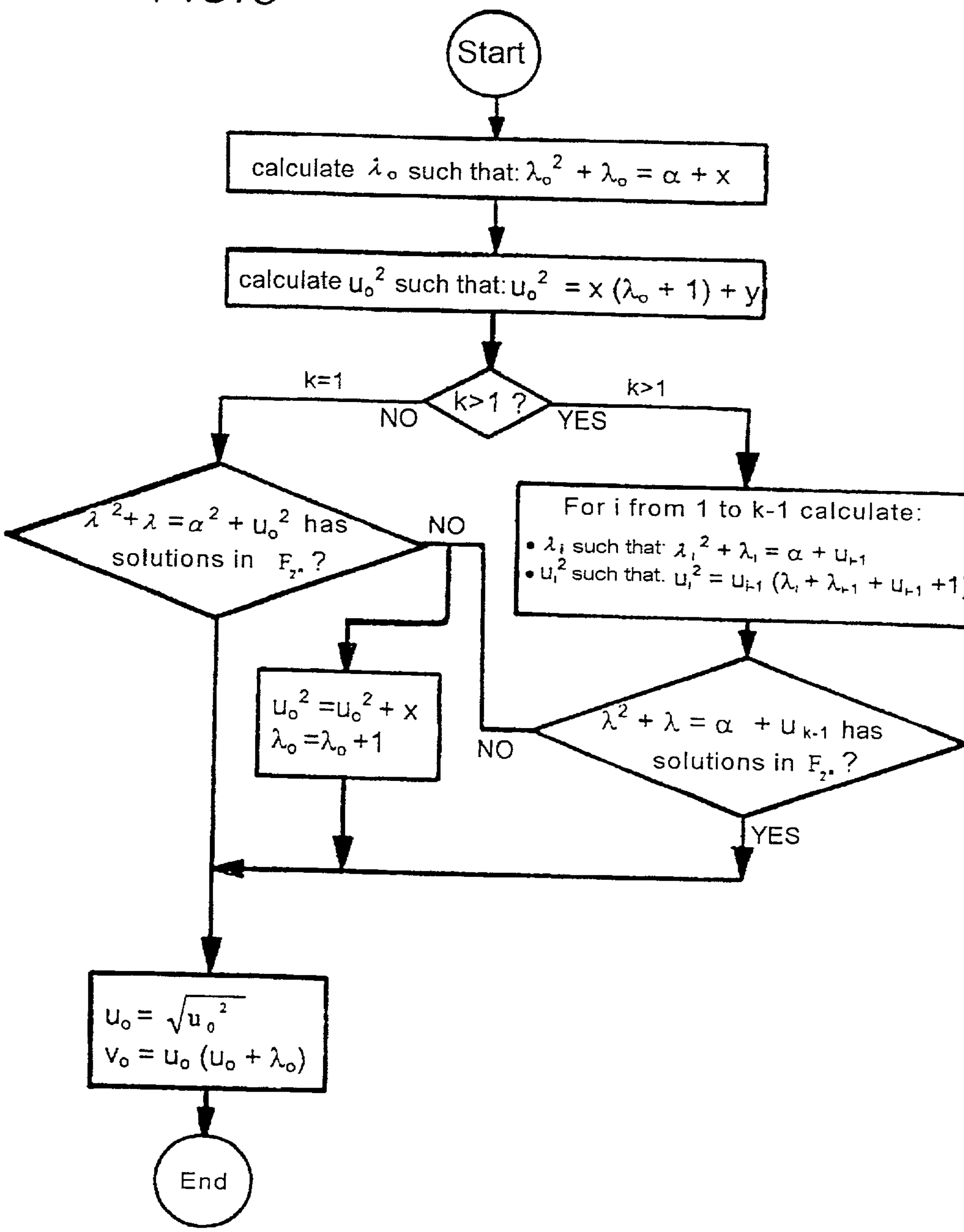


FIG. 3



Start
calculate λ_o such that: $\lambda_o^2 + \lambda_o = \alpha + x$
calculate u_o^2 such that: $u_o^2 = x(\lambda_o + 1) + y$
k>1 ?
k=1
NO
k>1
YES
$\lambda^2 + \lambda = \alpha^2 + u_o^2$ has solutions in F_{2^n} ?
NO
For i from 1 to k-1 calculate:
• λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$
• u_i^2 such that $u_i^2 = u_{i-1}(\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$
$u_o^2 = u_o^2 + x$
$\lambda_o = \lambda_o + 1$
NO
$\lambda^2 + \lambda = \alpha + u_{k-1}$ has solutions in F_{2^n} ?
YES
$u_o = \sqrt{u_o^2}$
$v_o = u_o(u_o + \lambda_o)$
End

FIG. 4

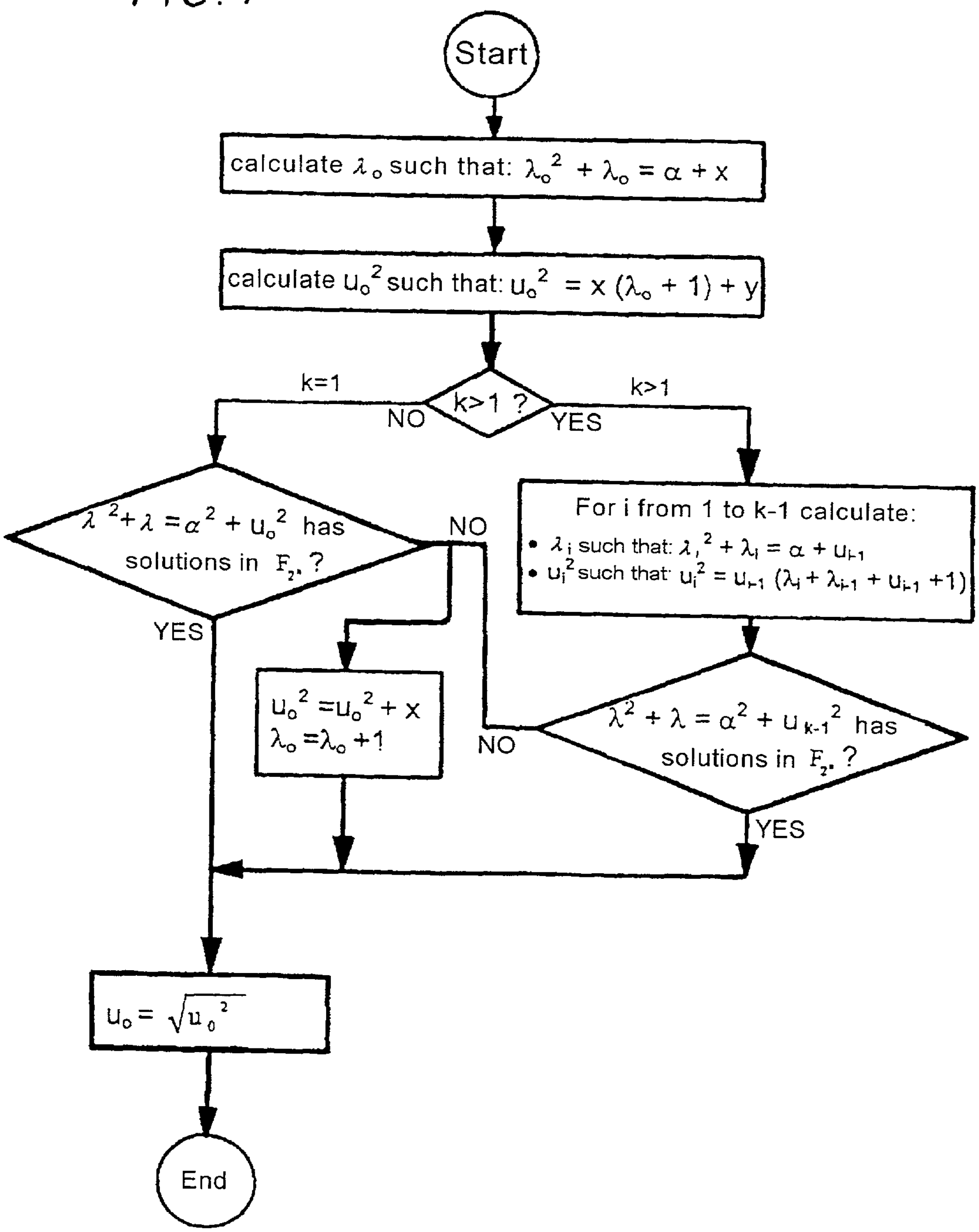

Start
calculate λ_o such that: $\lambda_o^2 + \lambda_o = \alpha + x$
calculate u_o^2 such that: $u_o^2 = x(\lambda_o + 1) + y$
k>1 ?
k=1
NO
k>1
YES
$\lambda^2 + \lambda = \alpha^2 + u_o^2$ has solutions in F_2. ?
NO
YES
For i from 1 to k-1 calculate:
• λ_i such that: $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$
• u_i^2 such that: $u_i^2 = u_{i-1}(\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$
$u_o^2 = u_o^2 + x$
$\lambda_o = \lambda_o + 1$
NO
$\lambda^2 + \lambda = \alpha^2 + u_{k-1}^2$ has solutions in F_2. ?
YES
$u_o = \sqrt{u_o^2}$
End

FIG. 5

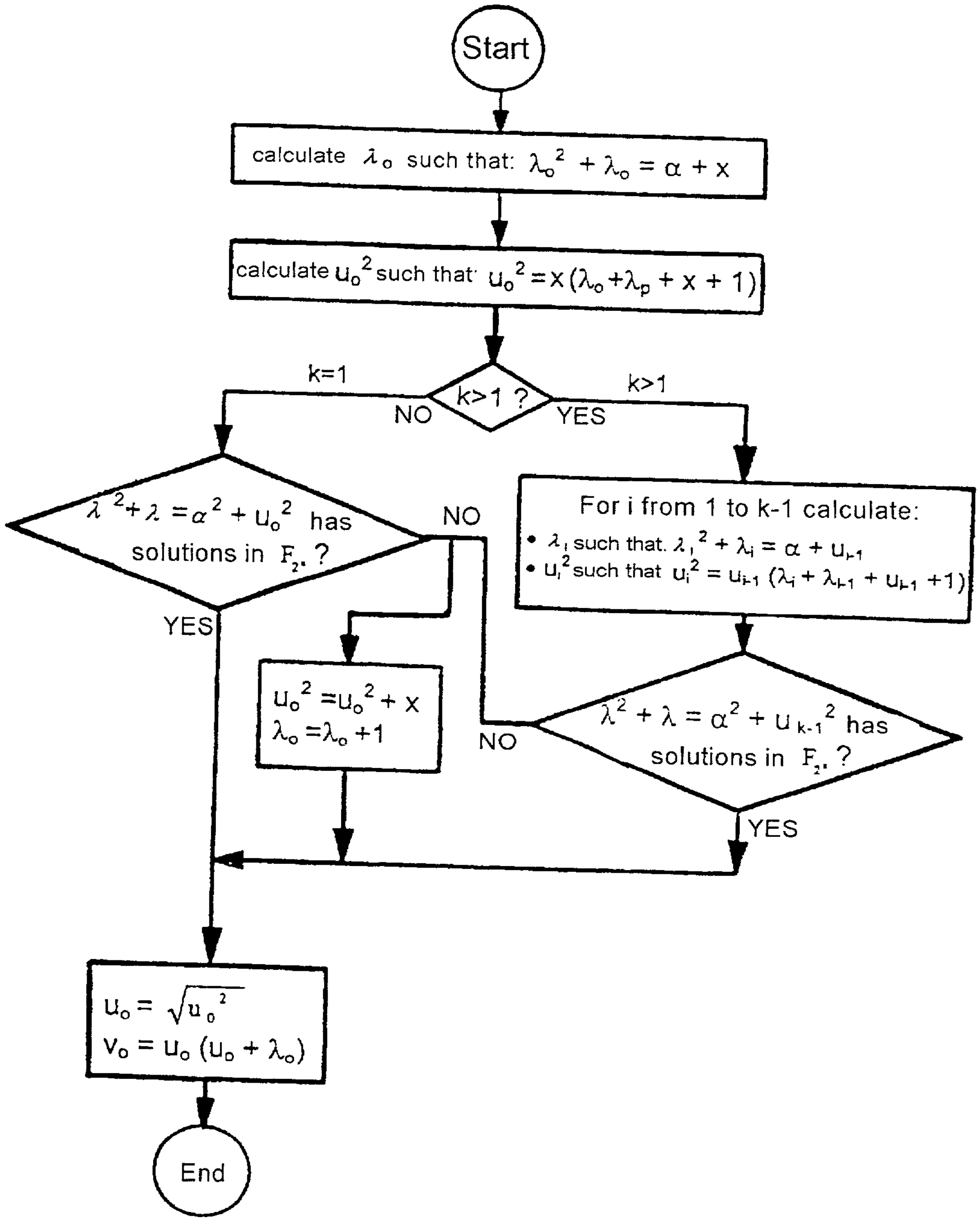

Start
calculate λ_o such that: $\lambda_o^2 + \lambda_o = \alpha + x$
calculate u_o^2 such that $u_o^2 = x(\lambda_o + \lambda_p + x + 1)$
k>1 ?
k=1
NO
k>1
YES
$\lambda^2 + \lambda = \alpha^2 + u_o^2$ has solutions in F_{2^n} ?
NO
For i from 1 to k-1 calculate:
• λ_i such that $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$
• u_i^2 such that $u_i^2 = u_{i-1}(\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$
YES
$u_o^2 = u_o^2 + x$
$\lambda_o = \lambda_o + 1$
NO
$\lambda^2 + \lambda = \alpha^2 + u_{k-1}^2$ has solutions in F_{2^n} ?
YES
$u_o = \sqrt{u_o^2}$
$v_o = u_o(u_o + \lambda_o)$
End

FIG. 6

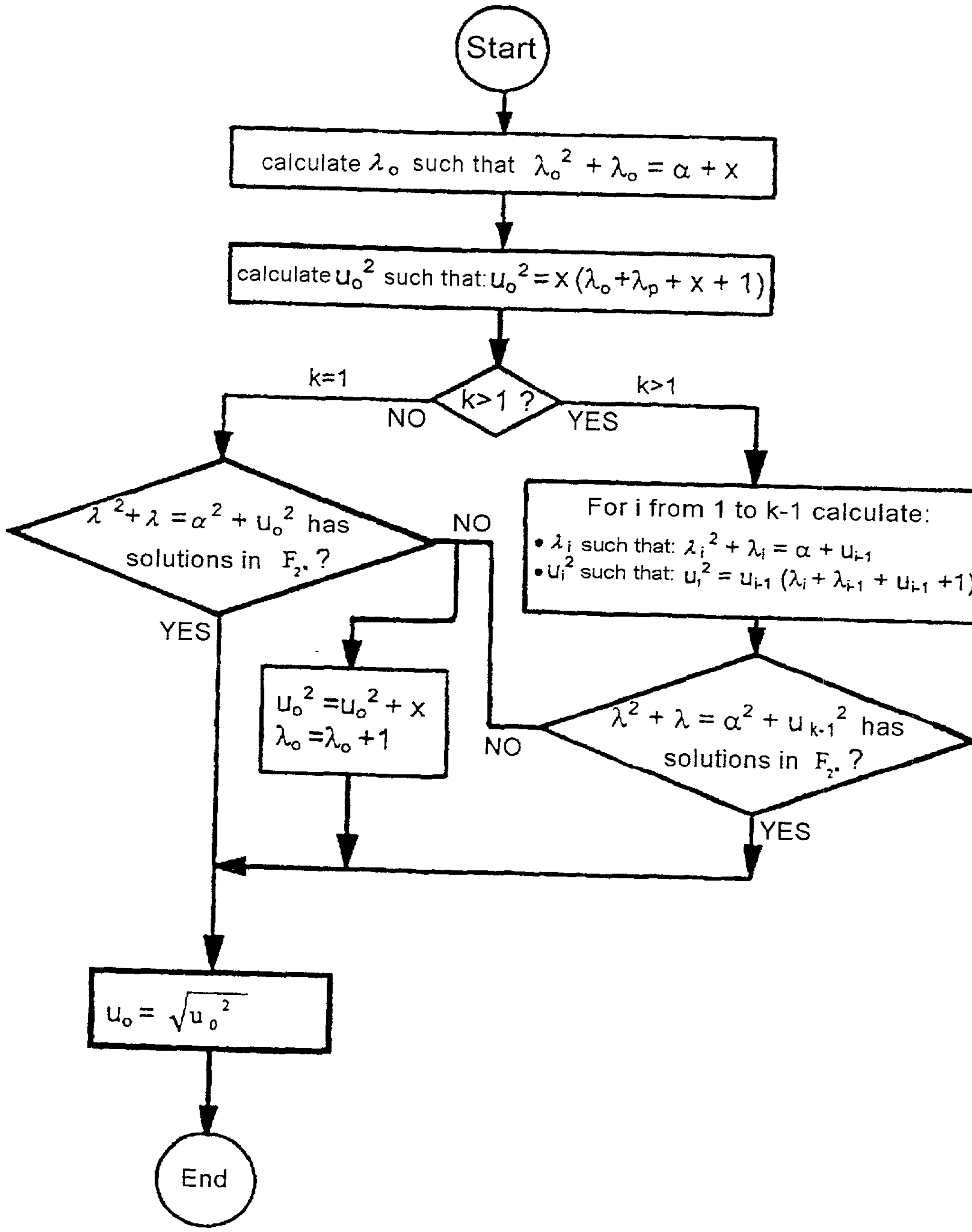
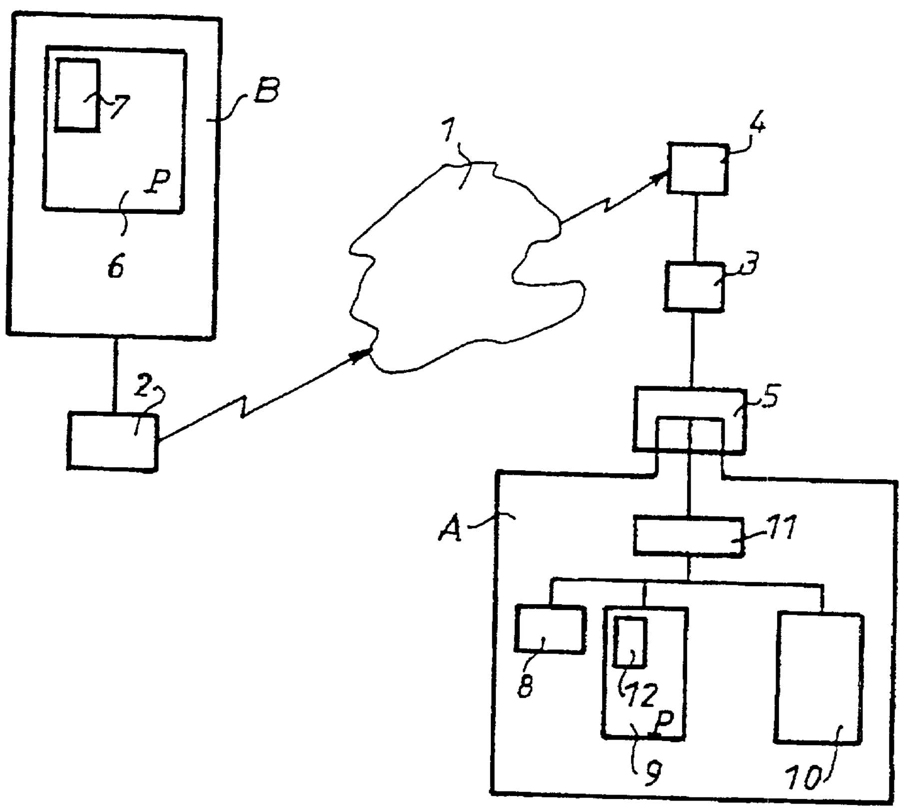

Start
calculate λ_o such that $\lambda_o^2 + \lambda_o = \alpha + x$
calculate u_o^2 such that: $u_o^2 = x(\lambda_o + \lambda_p + x + 1)$
k>1 ?
k=1
NO
k>1
YES
$\lambda^2 + \lambda = \alpha^2 + u_o^2$ has solutions in F_{2^n} ?
NO
YES
For i from 1 to k-1 calculate:
• λ_i such that: $\lambda_i^2 + \lambda_i = \alpha + u_{i-1}$
• u_i^2 such that: $u_i^2 = u_{i-1}(\lambda_i + \lambda_{i-1} + u_{i-1} + 1)$
$u_o^2 = u_o^2 + x$
$\lambda_o = \lambda_o + 1$
$\lambda^2 + \lambda = \alpha^2 + u_{k-1}^2$ has solutions in F_{2^n} ?
NO
YES
$u_o = \sqrt{u_o^2}$
End

FIG. 7



Combined Declaration for Patent Application and Power of Attorney

As a below-named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name; and that I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled "Calculation method for elliptic curve cryptography"

the specification of which (check one)

[ ] is attached hereto;

[ ] was filed in the United States under 35 U.S.C. §111 on ________________, as U.S. Appln. No. __________ *; or

[X] was/will be filed in the U.S. under 35 U.S.C. §371 by entry into the U.S. national stage of an international (PCT) application, PCT/FR00/01979 filed July 7, 2000, entry requested on ________________ *; national stage application received U.S. Appln. No. __________ *; §371/§102(e) date ________________ * (* if known)

and was amended on ____________________________________ (if applicable).

(include dates of amendments under PCT Art. 19 and 34 if PCT)

I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by any amendment referred to above; and I acknowledge the duty to disclose to the Patent and Trademark Office (PTO) all information known by me to be material to patentability as defined in 37 C.F.R. §1.56.

I hereby claim foreign priority benefits under 35 U.S.C. §§ 119 and 365 of any prior foreign application(s) for patent or inventor's certificate, or prior PCT application(s) designating a country other than the U.S., listed below with the "Yes" box checked and have also identified below any such application having a filing date before that of the application on which priority is claimed:

(Number)	(Country)	(Day Month Year Filed)	YES	NO
9908949	FRANCE	09/07/1999	[X]	[]
			[]	[]

I hereby claim the benefit under 35 U.S.C. §120 of any prior U.S. non-provisional application(s) or prior PCT application(s) designating the U.S. listed below, or under §119(e) of any prior U.S. provisional applications listed below, and, insofar as the subject matter of each of the claims of this application is not disclosed in such U.S. or PCT application in the manner provided by the first paragraph of 35 U.S.C. §112, I acknowledge the duty to disclose to the PTO all information as defined in 37 C.F.R. §1.56(a) which occurred between the filing date of the prior application and the national filing date of this application:

(Application No.)	(Day Month Year Filed)	(Status: patented, pending, abandoned)
PCT/FR00/01979	07/07/2000	pending

As a named inventor, I hereby appoint the following registered practioners to prosecute this application and to transact all business in the Patent and Trademark Office connected therewith:

All of the practioners associated with Customer Number 001444

Direct all correspondence to the address associated with **Customer Number 001444**; i.e.,

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(202) 628-5197

The undersigned hereby authorizes the U.S. Attorneys or Agents appointed herein to accept and follow instructions from ________________________ as to any action to be taken in the U.S. Patent and Trademark Office regarding this application without direct communication between the U.S. Attorneys or Agents and the undersigned. In the event of a change of the persons from whom instructions may be taken, the U.S. Attorneys or Agents appointed herein will be so notified by the undersigned.

Atty. Docket:

Title: "Calculation method for elliptic curve cryptography"

U.S. Application filed ____________, Serial No. ____________

PCT Application filed on July 7, 2000, Serial No. PCT/FR00/01979

I hereby further declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under 18 U.S.C. §1001 and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

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FULL NAME OF SECOND JOINT INVENTOR	INVENTOR'S SIGNATURE	DATE
RESIDENT	CITIZENSHIP	
POST OFFICE ADDRESS		

FULL NAME OF THIRD JOINT INVENTOR	INVENTOR'S SIGNATURE	DATE
RESIDENT	CITIZENSHIP	
POST OFFICE ADDRESS		

FULL NAME OF FOURTH JOINT INVENTOR	INVENTOR'S SIGNATURE	DATE
RESIDENT	CITIZENSHIP	
POST OFFICE ADDRESS		

FULL NAME OF FIFTH JOINT INVENTOR	INVENTOR'S SIGNATURE	DATE
RESIDENT	CITIZENSHIP	
POST OFFICE ADDRESS		

FULL NAME OF SIXTH JOINT INVENTOR	INVENTOR'S SIGNATURE	DATE
RESIDENT	CITIZENSHIP	
POST OFFICE ADDRESS		

FULL NAME OF SEVENTH JOINT INVENTOR	INVENTOR'S SIGNATURE	DATE
RESIDENT	CITIZENSHIP	
POST OFFICE ADDRESS		

ALL INVENTORS MUST REVIEW APPLICATION AND DECLARATION BEFORE SIGNING. ALL ALTERATIONS MUST BE INITIALED AND DATED BY ALL INVENTORS PRIOR TO EXECUTION. NO ALTERATIONS CAN BE MADE AFTER THE DECLARATION IS SIGNED. ALL PAGES OF DECLARATION MUST BE SEEN BY ALL INVENTORS.